CHAPTER 14
CW AND FM RADAR

William K. Saunders
Formerly of Harry Diamond Laboratories

14.1 INTRODUCTION AND ADVANTAGES OF CW

The usual concept of radar is a pulse of energy being transmitted and its round-trip time being measured to determine target range. Fairly early it was recognized that a continuous wave (CW) would have advantages in the measurement of the doppler effect and that, by some sort of coding, it could measure range as well.

Among the advantages of CW radar are its apparent simplicity and the potential minimal spread in the transmitted spectrum. The latter reduces the radio interference problem and simplifies all microwave preselection, filtering, etc. A corollary is the ease in the handling of the received waveform, as minimum bandwidth is required in the IF circuitry. Also, with solid-state components peak power is usually little greater than average power; CW then becomes additionally attractive, particularly if the required average power is within the capability of a single solid-state component.

Another very apparent advantage of CW (unmodulated) radar is its ability to handle, without velocity ambiguity, targets at any range and with nearly any conceivable velocity. With pulse doppler or moving-target indication (MTI) radar this advantage is bought only with considerable complexity. An unmodulated CW radar is, of course, fundamentally incapable of measuring range itself. A modulated CW radar has all the unwanted compromises, such as between ambiguous range and ambiguous doppler, that are the bane of coherent pulsed radars. (See Chaps. 15 to 17.)

Since CW radar generates its required average power with minimal peak power and may have extremely great frequency diversity, it is less readily detectable by intercepting equipment. This is particularly true when the intercepting receiver depends on a pulse structure to produce either an audio or a visual indication. Police radars and certain low-level personnel detection radars have this element of surprise. Even a chopper receiver, in the simplest video version, may not give warning at sufficient range to prevent consequences.

It should not be concluded that CW radar has all these advantages without corresponding disadvantages. Spillover, the direct leakage of the transmitter and its accompanying noise into the receiver, is a severe problem. This was recognized fairly early by Hansen[1] and Varian[2] and others. In fact, the history of CW radar shows a continuous attempt to devise ingenious methods to achieve the desired sensitivity in spite of spillover.

14.2 DOPPLER EFFECT

Complete descriptions of the doppler phenomenon are given in most physics texts, and a discussion emphasizing radar is to be found in Skolnik[3] (chap. 3, pp. 68–69).

When the radar transmitter and receiver are colocated, the doppler frequency f_d obeys the relationship

$$f_d = \frac{2v_r f_T}{c}$$

where f_T = transmitted frequency
c = velocity of propagation, 3×10^8 m/s
v_r = relative (or radial) velocity of target with respect to radar

Thus when the relative velocity is 300 m/s, the doppler frequency at X band is about 20 kHz. Alternatively, 1 ft/s corresponds to 20 Hz at this frequency. Scaling is a convenient way to handle other microwave frequencies or velocities.

As in a pulse radar, a CW radar that uses a rapid rate of frequency modulation, in order to sample the doppler, must have this rate twice the highest expected doppler frequency if an unambiguous reading is to be obtained. If the rate falls below the doppler frequency itself, there are problems of blind speeds as well as ambiguities. (A blind speed is defined as a relative velocity that renders a target invisible.) This will be discussed in more detail in Sec. 14.10; see also Sec. 15.3.

14.3 UNMODULATED CW RADAR

Spectral Spreading. The following discussion concerns the larger CW radars used for target illumination in semiactive systems, for acquisition, or for warning. A highly simplified diagram is shown in Fig. 14.1. Insofar as the primary operation of the equipment is concerned, the transmitted signal may be considered an unmodulated CW, although small-amplitude amplitude modulation (AM) or small-deviation frequency modulation (FM) is sometimes employed to provide coding or to give a rough indication of range. The modulation frequency is chosen to lie above the doppler band of interest, and the circuitry is designed to degrade the basic noise performance as little as possible.

A spectrum-spreading problem is also posed by conical scan. In this case, the scan frequency will lie, it is hoped, below any doppler of interest, with the result that the conical-scan frequency will appear as small-amplitude sidebands on the doppler frequency when it is recovered in the equipment. In the material that follows, these secondary issues will be largely ignored, and an unmodulated CW transmission plus a receiver that introduces no intentional modulation will be assumed.

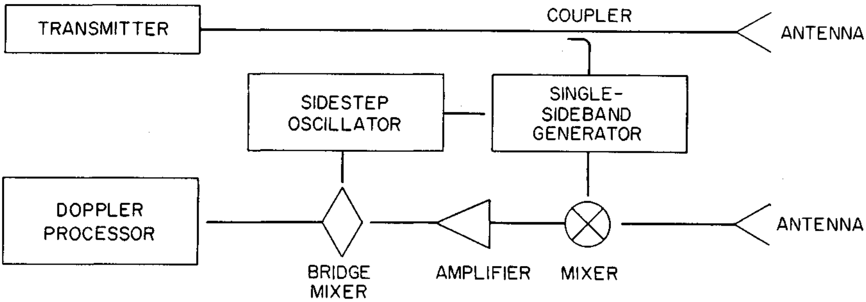


FIG. 14.1 Basic diagram of a CW radar.

Noise in Sources. The primary noise problem is in the microwave source itself. All klystrons, triodes, solid-state sources, etc., generate measurable noise sidebands in a band extending beyond any conceivable doppler frequency. Unless the source is unacceptably bad, these noise sidebands may be subdivided into pairs: those whose phase relationship to each other and the main carrier line is such as to represent amplitude modulation and those corresponding to a small index of frequency or phase modulation. The AM components, offset a given frequency from the carrier, are usually many decibels below the corresponding FM components. Moreover, balanced mixers, limiters, and other design techniques may be used to suppress AM noise. FM noise therefore is usually of greater concern in CW radar.

The FM noise of a good klystron amplifier driven by a klystron oscillator having either an active or a passive FM stabilizer is about 133 dB below the carrier in a 1-Hz bandwidth* 10 kHz removed from the carrier. The noise power decreases approximately as $1/f^2$ at larger offsets. The corresponding AM noise is 150 to 160 dB below the carrier. As the local-oscillator signal of a CW radar is usually derived from the transmitter, it too will show comparable amounts of noise. With noise this low relative to the carrier there is no concern that a significant portion of the target's energy will be lost in any practical filtering operation. What is of concern is the energy of the noise components carried into the receiver on the unwanted spillover and clutter signals. Moreover, should vibrations cause variations in the length or lengths of the spillover path the problem is more severe, since such effects introduce spectral lines that may fall directly into the doppler band of interest.

Noise from Clutter. Clutter, unwanted reflections from the ground, rain, etc., reflects the transmitted power and its noise sidebands back to the receiver. Suppose that in the foreground there is an industrial area having a clutter cross section of 0.1 m^2 per square meter of ground illuminated. Consider only an area located 2½ to 3½ km from the radar ($R \approx 35$ dB, where the range R is in decibels relative to 1 m) and illuminated with an antenna having 0.1 rad of beamwidth. There is a reflecting cross section σ of approximately

*In this chapter a 1-Hz bandwidth has been chosen as the reference. There has been no consistent practice in the literature, since it has been customary to use either the bandwidth of the system in question or that of the measuring equipment.

$$3 \times 10^3 \times 0.1 \times 10^3 \times 0.1 = 3.0 \times 10^4 \text{m}^2 \text{ or } 45 \text{ dB}$$

Suppose that there is present a target at a greater range and that a +22-dB signal-to-noise ratio in a 1-kHz bandwidth is needed to have a suitable probability of detection with an acceptable probability of false alarm (this includes allowance for a +10-dB receiver noise figure). If the target produces a 10-kHz doppler signal, the noise of the clutter signal must not exceed −144 dBm* + 22 dB = −122 dBm at the 10-kHz offset. If a transmitter is assumed in which the noise sidebands in a 1-kHz bandwidth are 103 dB below the carrier, the clutter signal must not exceed −19 dBm. Assume that the antenna gain is +30 dB and the transmitted power +60 dBm at X band. We have

$$\begin{array}{lccccccc} & \text{(power)} & (G^2) & (\lambda^2) & (64\pi^3) & (\sigma) & (R^4) \\ -19 \text{ dBm} > & +60 \text{ dBm} & +60 \text{ dB} & -30 \text{ dB} & -33 \text{ dB} & +45 \text{ dB} & -140 \text{ dB} \\ > & \multicolumn{6}{l}{-38 \text{ dBm, or a 19 dB favorable margin}} \end{array}$$

It should be noted that we have assumed a very quiet transmitter and clutter centered about a 3-km range. (This ignores correlation; see below.)

It is convenient to express the FM-noise sidebands on the transmitter in another manner. Consider a single modulating frequency, or line, f_m in the noise having a peak frequency deviation ΔF_p. By the frequency-modulation formulas, the carrier has a peak amplitude of $J_0(\Delta F_p/f_m)$, and each of the nearest sidebands a peak amplitude of $J_1(\Delta F_p/f_m)$.[4] If the arguments are small, as they must be for our computations, the Bessel functions may be approximated by

$$J_0(X) \sim 1$$
$$J_1(X) \sim \frac{X}{2}$$

Hence the power ratio between the carrier and one of the first harmonic sidebands is $\Delta F_p^2/4f_m^2$. For the ratio of the power in both sidebands to that in the carrier (the quantity usually of interest),

$$\text{FM noise power} = \frac{\Delta F_p^{\ 2}}{2f_m^{\ 2}} = \frac{\Delta F_{\text{rms}}^{\ 2}}{f_m^{\ 2}} \tag{14.1}$$

A 1-Hz peak deviation at a 10-kHz rate represents a double-sideband noise ratio of $\frac{1}{2}(\frac{1}{10^4})^2$ or −83 dB with respect to the carrier. The −103-dB (double sideband at 10 kHz) transmitter used as an example above has a peak deviation of 0.1 Hz. These numbers are all equivalent only when referred to a particular bandwidth, 1 kHz in this case. (The description in hertz is convenient only when the noise is expressed in the bandwidth of interest in a particular radar.)

The concept of a peak signal is properly associated only with a sine wave. With random noise the rms description is more meaningful. However, a noise power at a given frequency and bandwidth is equivalent to that which would be produced by a sine wave having a certain peak or rms value.[5]

To carry the computations further, the correlation effect must be discussed.

*Thermal noise in a 1-kHz band at 20°C.

When a transmitter is producing FM noise, it may be thought to be modulating in frequency at various rates and small deviations. Consider, for example, a particular one of these modulating frequencies. If it is a low frequency and the delay associated with the spillover or clutter is short, the returning signal finds the carrier at nearly the same frequency that it had at the time of transmission; that is, the decorrelation is small. Higher frequencies in the noise spectrum have greater decorrelation. Moreover, the effect is periodic with range: For any given sinusoidal modulating frequency the FM noise produced will increase as a function of range out to a given range and will then decrease. The zeros occur at the ranges $R = nc/2f_m$, where f_m is the frequency of the sinusoidal modulating component, c is the velocity of light, and n is any integer.

However, in general, one deals with noise rather than a sinusoidal component. For this reason, the discrete zeros indicated are seldom of interest, and for ease in the computations the signal is assumed to be decorrelated at a frequency f_l approaching $f_l = c/8R$. Some frequencies higher than f_l cause no problems at particular ranges, but nearby ones do so. Moreover, as the formulas below will show, the deviation of the recovered signal can be twice as great as that of the transmitter. (The returning signal may be swinging up while the transmitter is swinging down.)

The peak voltage of the first harmonic sideband in the IF spectrum of an FM signal mixed with itself after a time delay $T = 2R/c$ is[6]

$$v_{p1} = J_1\left(2\frac{\Delta F_p}{f_m} \sin \pi f_m T\right)$$

and the peak voltage of the carrier is

$$v_{p0} = J_0\left(2\frac{\Delta F_p}{f_m} \sin \pi f_m T\right)$$

In both formulas ΔF_p is the peak frequency deviation of the carrier. As before, $J_1(X) \sim x/2$, $J_0(X) \sim 1$, $X<1$. Hence the ratio of the power in a single sideband to that in the carrier is

$$\frac{P_s}{P_c} = \left(\frac{\Delta F_p}{f_m} \sin \pi f_m T\right)^2$$

and in the pair of sidebands

$$\frac{P_{2s}}{P_c} = 2\left(\frac{\Delta F_p}{f_m} \sin \pi f_m T\right)^2 \qquad (14.2)$$

The maximum value of this is $2(\Delta F_p/f_m)^2$, which is in agreement with the maximum deviation of the IF being twice as large as that of the transmitted frequency [Eq. (14.1)].

For smaller values of $f_m T$ the double-sideband power ratio is

$$\frac{P_{2s}}{P_c} = 2(\pi \Delta F_p T)^2 \qquad \pi f_m T < 1 \qquad (14.3)$$

This is an interesting formula as it shows that when ΔF_p is constant, as it is with many klystrons, the correlated noise power is independent of frequency and directly dependent on range.

A convenient curve which gives the ratio of noise at the receiver to measured noise on the transmitter (Fig. 14.2) is based on the approximations of formulas (14.2) and (14.3). The dotted portion of the curve reflects only the approximations formula (14.2).

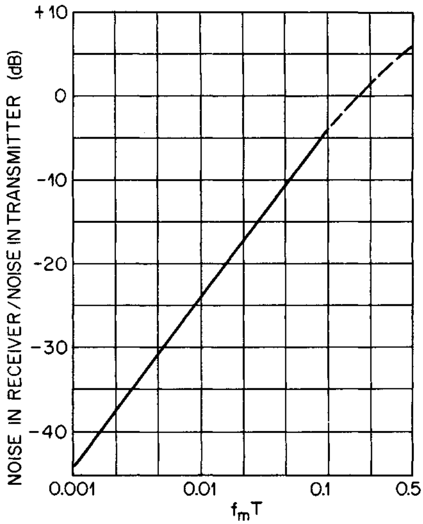


FIG. 14.2 Noise suppression by the correlation effect.

The problem started above can now be completed. The center of the clutter was taken at 3 km or a T of 2×10^{-5} s. The frequency of interest was 10^4 Hz. Hence $f_m T = 0.2$, which is beyond the region of noise correlation.

An unmodulated CW radar must contend with clutter almost down to zero range. Without the correlation effect, this would generally be impossible. For a given antenna beam the width of the illuminated clutter area decreases as R but there is a $1/R^4$ in the radar equation. The result is that the clutter return varies as $1/R^3$. The correlation effect shows that for a fixed noise frequency the correlated noise sidebands decrease as R^2 [Eq. (14.3), with $T = 2R/c$]. Hence there is an apparent rate of increase of $1/R$. The integral representing the clutter power appears to diverge, but two factors so far ignored have a decisive influence at very short ranges. The first is that the intersection of a beam emitted from an antenna of finite height and the earth is the interior of a hyperbola and not a sector, as implied above. The second is that at close ranges the clutter is in the Fresnel region of the antenna and the far-field gain formula no longer applies. In a more careful analysis, by using either of these factors, the integral may be shown to be convergent.

Shreve[7] has derived a formula for the double-sideband noise power. He took the boundary of the Fresnel region $R_F = D^2/\lambda$ as the lower limit of the integral.

His formula for the above parameters yields a value of −117 dBm for the correlated noise power from the clutter. This is for the extreme case of a single antenna at exact ground level looking into very severe clutter (0.1 m^2/m^2).

A more practical way to look at the problem is to note that clutter from very short ranges and spillover are almost equivalent phenomena. For a ground-based CW radar to operate at maximum sensitivity, two antennas must be employed; this reduces both the spillover and the near-in clutter since no close-in point can be in the main lobes of both transmitting and receiving beams. Moreover, as described below, spillover cancellation (and hence near-in clutter cancellation) is usually employed.

The discussion above assumes that the local oscillator employed in the radar is either derived from the transmitter or locked to it with a servo which has a frequency response sufficiently high to cover the doppler and noise band of interest.

Microphonism. Microphonism can cause the appearance of additional noise sidebands on the spillover and occasionally on the clutter signals. If the structures are sufficiently massive, the microphonism is greatest at the lower frequencies, where it can be counteracted by a feedthrough servo. To this end, however, it is most important that microwave components employed in the feedthrough nulling as well as in the remainder of the microwave circuitry be as rigid as possible.[8] It is customary to use a milled-block form of construction. In the rare cases where a single antenna plus duplexer or a pair of nested antennas has been used in an airborne high-power CW radar, the mechanical design problems have been all but insurmountable. Even in a ground-based radar, fans, drive motors, motor-generator sets, rotary joints, cavitation in the coolant, etc., are very troublesome.

Scanning and Target Properties. In addition to the spectral spreading caused by transmitter noise and by microphonism, there is a spreading of the CW energy by the target and by the scanning of the antenna. Generally, the spreading by even a rapidly scintillating aircraft target does not produce appreciable energy outside a normal doppler frequency bandwidth. The filter is usually set by the acquisition problem or the time on target rather than by the intrinsic character of the return signal. Rapid antenna scan, however, can cause an appreciable broadening of the spectrum produced by the clutter. Were it not for the particular shape of the typical antenna beam, the transients produced by clutter while scanning would be far more serious.

An approximate analysis assumes a gaussian two-way gain for the antenna, $G^2 = e^{-2.776\theta^2/\theta_B^2}$, where θ is measured from the axis of the beam and θ_B is the beamwidth between the half-power points of the antenna. We shall discuss a two-way pattern down 3 dB at $\pm\frac{1}{2}°$ ($\theta_B = 1°$). If the antenna scans 180° a second, we shall need the Fourier transform of e^{-at^2} with $a = 9 \times 10^4$. This has the form $Ae^{-\omega^2/3.6\times10^5}$, which is down to $\frac{1}{1000}$ (60 dB) of its peak when $\omega^2/(3.6 \times 10^5) = 6.9$, $\omega \approx 1575$, and $f \approx 250$ Hz.

Actual antenna patterns produce somewhat less favorable transients than the gaussian shape. Limiting in the receiver is equivalent to altering the shape of the beam.[9] For any antenna pattern there is a definite limitation on the scanning speed of a narrow-beam antenna. Actually, mechanical limitations usually prevent trouble except with the very slowest targets, but with nonmechanical scanning methods degradations may occur.

14.4 SOURCES

Master Oscillator Power Amplifier (MOPA) Chains. Requirements peculiar to CW radar are the use of extremely quiet tubes throughout the transmitter chain, very quiet power supplies, and, often, stabilization to reduce the total noise of the system. In theory, any of the methods for measuring FM and AM noise to be discussed in Sec. 14.5 might be modified to produce a noise-quieting servo. Practical considerations have resulted in a wide variety of additional schemes. The simplest is the introduction of a high-Q cavity between a klystron driver and the power amplifiers. Q's of 20,000 to 100,000 are normally employed. The action of the cavity is primarily that of an additional reactive element directly in parallel with the cavity in the klystron. With a high-quality reflex klystron having an FM noise 110 dB below the carrier in a 1-Hz band spaced 10 kHz from the carrier, use of the high-Q cavity as a passive stabilizer reduces the corresponding FM noise 130 to 135 dB below the carrier. The cost is a power loss of about 11 dB. The 20 to 25 dB noise improvement is obtained at most frequencies of interest. No noticeable improvement is made in the AM noise level at doppler frequencies by this technique.

It should be remembered that this results only in a stable driver, any noise generated in the power amplifier being unaffected. And, as noted above, unless the local oscillator is generated from the output of the power amplifier, which, of course, cannot be done in a pulse doppler system, this noise is uncorrelated. Fortunately, good power amplifiers driven by highly regulated supplies add extremely small amounts of excess, or additive, noise. (See Fig. 14.8.)

It is to be noted that the illuminator for the basic Hawk surface-to-air missile system used a magnetron as the transmitter rather than a MOPA chain. Cost, availability, lower weight, and lower high-voltage requirements were all factors in the choice. Later versions of Hawk use the inherently quieter klystrons. Banks[10] provides a definitive overview of the Hawk illuminator including both the noise degeneration loop (feedthrough nulling) and the transmitter stabilization microwave circuitry. An interesting feature of the latter is a spherical cavity that is far stiffer under vibration than the usual cylindrical cavities.

Active Stabilization. All the schemes for active stabilization on a MOPA chain depend on the use of a high-Q cavity as the reference element. The cavity must be isolated from the tube so that it functions as a measuring device without introducing the pulling that results from the frequency dependence of its susceptance.

For reflection and transmission cavities, useful equations adapted from Grauling and Healy[11] are given below. For a matched reflection cavity,

$$\Gamma = \frac{Z-1}{Z+1} = \frac{j\delta Q}{1+j\delta Q} = \frac{j2\delta Q_L}{1+j2\delta Q_L}$$

where Γ = reflection coefficient
$\delta = (f - f_0)/f_0$
Q = unloaded Q of cavity
Q_L = loaded Q of cavity
Z = normalized impedance looking into cavity

The transmission cavity has similar characteristics except that both the carrier (V_c) and sidebands (V_{sb}) are passed. For a transmission cavity,

$$V_0 = \frac{V_1}{3 + j2\delta Q} = V_c + V_{sb}$$

(Both coupling coefficients have been assumed equal to unity.) V_1 is the input voltage and V_0 the output voltage. With a little algebra, it is seen that the frequency-dependent terms of Γ and V_{sb} are similar in form. In stabilization systems, $f - f_0$ is kept small and

$$\text{Reflection:} \qquad \Gamma \approx j2\delta Q$$

$$\text{Transmission:} \qquad V_{sb} \approx 2(-2j\delta Q)$$

One might expect that the stabilization would be equally effective in reducing regardless of the frequency. This ignores two factors: the cavity has only a finite linear range, and larger values of f_m may produce sidebands that lie outside this range; for stability the servo that follows the cavity must have a response that rolls off at the higher frequencies.

The simplest stabilization bridges result directly from the character of the transmission and reaction cavities. The transmission-cavity bridge might have the arrangement of Fig. 14.3.

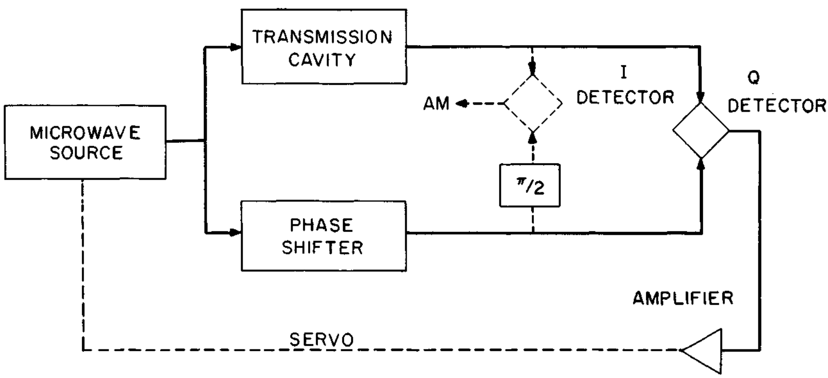


FIG. 14.3 Transmission bridge, video version.

The phase shifter is set so that the Q, or quadrature, detector receives signals in phase quadrature* and, to first order, is sensitive only to FM. Should there be a requirement for AM stabilization, it would normally be only at very low frequencies such as those introduced by the power supplies. Such a requirement could be met easily by adding a $\pi/2$ phase shift and an I (coherent amplitude-sensitive) detector, as indicated by the dotted blocks in Fig. 14.3. The discussion of the servo constants will be postponed until the end of this section.

*The most sensitive technique for adjusting the quadrature detector is to introduce intentional AM and null this by adjustment of the phase shifter. Maximizing the response to FM yields a less exact adjustment.

An obvious disadvantage of the transmission-cavity bridge is that the carrier is not suppressed in the microwave circuitry. Since the total input power is limited by fear of crystal damage and of exceeding the linear range in the mixing process, the intelligence signal power at a relatively low level is in competition with the thermal noise generated by the crystals.

The reflection cavity has an advantage in that a sizable portion of the carrier power is absorbed if the transmitter is kept tuned to the frequency of the cavity. This eliminates much of the saturation problem.

A particularly attractive arrangement was proposed by Marsh and Wiltshire[12] (Fig. 14.4). It is the basis of the earliest successful FM noise-measuring instruments and has been employed in stabilization as well.[13] It is the only bridge that removes most of the carrier power to avoid saturation of the mixer crystals. The key to its operation is a balancing element which matches exactly the reflection from the cavity at resonance.

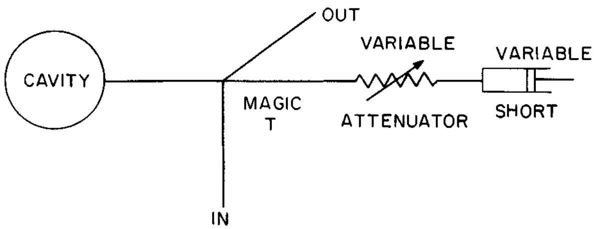


FIG. 14.4 Marsh and Wiltshire microwave bridge.

At resonance the cavity is nearly a perfect absorber, and the residual reflection is tuned out with the variable short and variable attenuator. As the frequency varies, the cavity produces a reactive component which alters the balance. The result, at least for small deviations, is a double-sideband suppressed-carrier signal. With care, the carrier may be suppressed as much as 40 dB with manually or statically balanced bridges and as much as 60 dB if either the cavity or the source is electronically or thermally tuned. The result is that 2 W of power may be handled in the manually tuned version, and up to 1000 W in the servo-tuned equipment. The balance of the circuitry is shown in Fig. 14.5.

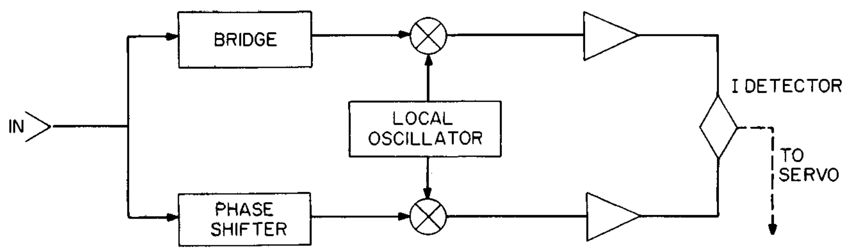


FIG. 14.5 IF circuitry for a Marsh and Wiltshire bridge (shown in Fig. 14.4).

As is seen, a properly phased second sample of the transmitter is processed in a parallel path and serves as the reinjected carrier to recover the sidebands at the *I* detector. Since the lower path is nondispersive and since the sidebands are small (a requisite for all the above quieting schemes), the large signal may be regarded as an essentially pure carrier in the reinjection process. The LO must be reasonably quiet, and the phase delay of the amplifiers must be matched.

Figure 14.6 gives the servo-loop gain, and Fig. 14.7 shows the three resulting curves: A, the FM noise on the free-running oscillator; B, the FM noise of the stabilized oscillator; and C, the expected theoretical improvement based on the servo gain and the noise analysis.

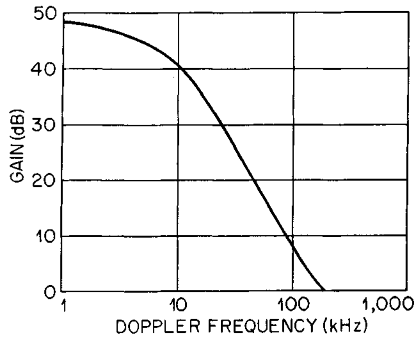


FIG. 14.6 Frequency control-loop gain.

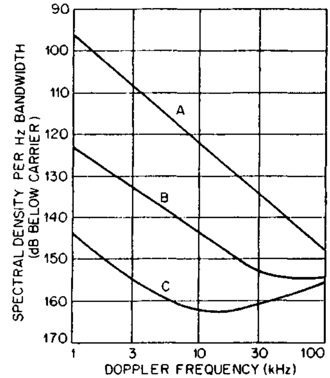


FIG. 14.7 Transmitter noise spectra: A, free-running FM; B, closed-loop FM; C, free-running FM divided by control-loop gain.

For details of the construction and a discussion of the results, see the first edition (1970) of the handbook or Ref. 12.

Stabilization of Power Oscillators. The active methods described above can be used for the stabilization of power oscillators as well as for the stabilization of drivers. The measurement bridges are unchanged, but servo circuits must be altered to operate at high voltage and, in some cases, supply considerable power, since multicavity klystrons and crossed-field devices such as magnetrons and amplitrons can be modulated only through their high-voltage high-current supplies. Moreover, the fact that these are essentially "stiff" devices imposes more stringent requirements on the design of the servo.

14.5 NOISE MEASUREMENT TECHNIQUE

Two basic types of noise measurement are of interest to the designer: primary-noise measurements to be made on drivers or power oscillators and additive-, or excess-, noise measurements to be made on amplifiers, multipliers, rotary joints, etc.

Although microwave cavities, such as used in the Marsh-Wiltshire bridge, were widely employed at one time, commercial instruments generally avoid them.[14] They accomplish this by comparing, in a phase detector, the source under test with either an external nearly duplicate source or an internal source supplied by a portion of the test equipment. If nearly duplicate sources are used, one is assured that at least one of them (not necessarily always the same one) is at least 3 dB quieter at each offset frequency than the phase noise indicated by the instrumentation. By using three essentially duplicate sources and measuring the phase noise generated by each pair at all the desired offset frequencies, one derives three sets of measurements. This leads to three equations with three unknowns, and the phase noise of each of the three sources can be derived as a function of frequency. If one of the internal sources supplied by the instrumentation is used, there is a distinct limitation owing to the phase-noise characteristic of that particular internal source. In general, this is the paramount limitation since the noise floor of the phase detection circuitry is usually well below that of the internal or, for that matter, the external reference sources. This assumes that the AM noise on both sources used in the test is well below the phase-modulation noise. The only safe course is first to measure the AM noise on any unknown source with a simple amplitude detector available with the instrumentation.

The instruments can provide a servo voltage to hold the two sources at the same frequency and in quadrature at the phase detector. If the sources are such that neither is readily voltage-tunable, then one source is chosen at a typical IF frequency away from the other, and an IF oscillator is locked, in the mean, to the difference frequency. This technique was originally employed in military test equipment designed to measure noise on radars in the field.[15,16] The instruments provide a wide range of internal frequencies through a combination of synthesizer techniques at the lower frequencies and combs going up to 18 GHz. The latter are created by using the harmonics of a step-recovery diode multiplier.

The signal coming from the phase detector is filtered to remove microwave frequencies and is then amplified in a low-noise baseband amplifier. The resulting phase noise can be measured by any of a variety of methods, including spectrum analyzers and analog wave analyzers. The most accurate and convenient method for the measurement of the lower frequency noise is the fast Fourier transform (FFT). The method is too time-consuming for analysis of the far-out phase noise.

With computer control of all of the components of the test equipment almost any desired measurement can be made, adjusted for filter shape, and printed out. There is even an option to remove spurs (spurious frequencies), occurring during the measurements, from the calculations and from the data. All this comes at a considerable cost, not all of which is monetary. One's ability to understand any laboratory technique and its inherent reliability are usually inversely related to complexity. For example, given enough equipment, each generating a plethora of internal signals, one is almost guaranteed spurs in any sensitive measurement. If the ultimate aim is a quiet transmitter, one attributes such spurs to gremlins in the test equipment at one's peril. A well-shielded screen room with a minimum of (well-understood) instrumentation in that screen room eliminates many variables.

In general, modern commercial instrumentation is vastly superior to the earlier cavity bridges for making routine measurements on low-power sources. It can measure phase noise very close to the carrier if the servo is tailored to force the two sources to track in the mean, without effectively locking them together, at the offset frequency of interest. By knowing the servo's characteristics, the instrumentation can adjust the data to reflect the phase noise actually present. The technique is limited, at small offset frequencies, by thermal noise that competes with the low values of phase deviation permitted by the servo. Even with this limitation, one is considerably better off than with a cavity bridge whose sensitivity falls off rapidly at small offsets. Without the aid of commercial phase-bridge instrumentation, it would have been difficult to develop the crystal sources having much reduced phase noise at close-in frequencies. (These have been key components in long-range airborne radars that are required to detect crossing targets immersed in clutter.) Nor are commercial instruments limited in their ability to measure phase noise at the larger offsets. They appear to have just two significant limitations. Cavity bridges are superior for development work on state-of-the-art sources, especially those that are difficult or expensive to produce in pairs, and in the measurement of high-power transmitters such as the Hawk illuminator. Compare curve *I* of Fig. 14.8, measured in the early 1960s, with curve *P* of the same figure, which is the measurement floor of typical commercial instrumentation.

An alternative to both cavity and source comparison techniques is the use of a delay line to provide a primary reference for the measurement of phase noise. A method that eliminates the noise contributed by the local oscillator is suggested in the appendix of Ref. 16. Unfortunately, the accuracy of any phase-noise measurement that depends on a delay line is proportional to the length of the delay. Long delays imply difficulty in maintaining sufficient signal amplitude to make satisfactory measurements. Incidentally, as noted above, many measurement techniques can be altered to provide a valid source-stabilization method. The delay line is an exception. When one attempts to servo-out phase noises at the

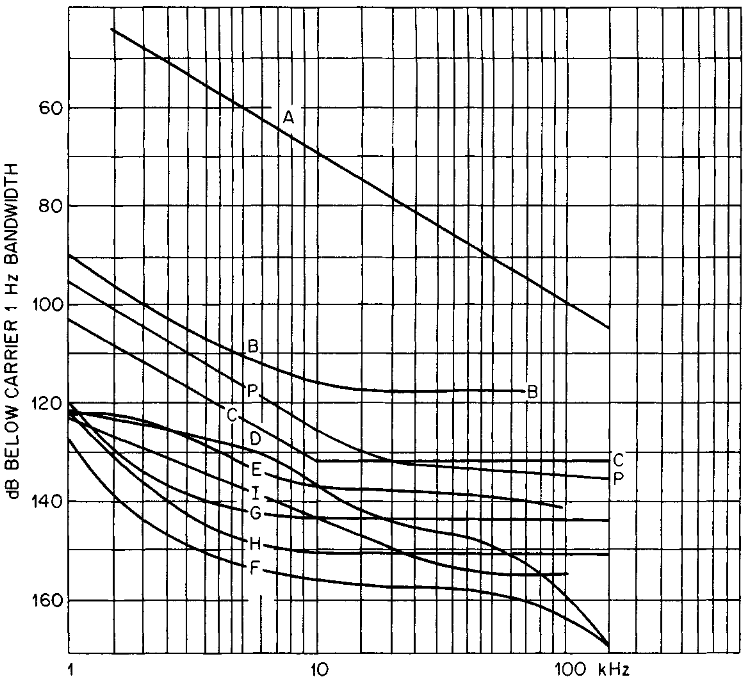


FIG. 14.8 FM noise in microwave sources. *A*, voltage-controlled *LC* oscillator multiplied to X band; *B*, crystal-controlled oscillator, step-recovery multiplier, to X band (*courtesy of D. Leeson*); *P*, noise floor at X band of 11729B/8640B combination (*courtesy of Hewlett-Packard*[14]); *C*, crystal oscillator (ST cut) multiplied to X band (*courtesy of Westinghouse Corporation*[18]); *D*, compact X-band klystron CW amplification (*Hughes*); *E*, compact X-band klystron pulsed amplification (*Hughes*); *F*, X-band klystron CW amplification (*Varian*); *G*, X-band klystron pulsed amplification (*Varian*); *H*, S-band electrostatically focused klystron amplifier (*Litton*); *I*, curve *B* in Fig. 14.7. Note that curves *D* to *H* are additive-noise measurements.

higher offset frequencies, one runs into the Nyquist restriction. For any fixed delay length, there is a corresponding offset frequency where the servo gain must go to zero or the whole system will become unstable.

Except for multipliers and dividers, the measurement of additive or excess phase noise* on components such as power amplifiers is considerably easier than the similar measurements on sources. All that is required is one moderately quiet source, a phase shifter, a phase detector, a suitable wave analyzer, and a method of calibration. The commercial instruments, described above, provide all this and much more. The reason that the source phase noise is not critical to the measurement is that it is common practice to add sufficient coaxial or microwave delay to equalize the two paths to the phase detector. Figure 14.2 indicates what such equalization (i.e., correlation) buys. As above, the amplitude modulation introduced by the source or the component under test must be checked first. For very demanding measurements, such as shown in curve *F* of Fig. 14.8, it might be well to consult the 1970 edition of this handbook and Ref. 17 of this chapter. For such work, a screen room is a must.

The measurement of multipliers is considerably more difficult since the two signals arriving at the phase detector must have the same frequency. This implies that two similar multipliers must enter the circuitry, and one has most of the problems associated with the measurement of sources. The only problem one is spared is the phase locking, which is usually required when working with sources. Fortunately, well-designed multipliers usually add little phase noise to a radar (above that to be expected from the increased FM deviation produced by the multiplication process). When 100 sources, consisting of a crystal oscillator plus a multiplier chain, were supplied by a subcontractor to a military radar program, the only ones that were unable to meet an extremely severe specification were ones that had substandard crystals in the oscillator.[16]

Because of the similarity of methods, measurement of noise in pulsed transmitters will merely be sketched. The measurement of pulsed sources is intrinsically much more difficult than the measurement of CW sources. The pulse structure produces very substantial AM that inevitably conflicts in direct and indirect ways with any attempt to measure the FM. In fact, it is possible to measure FM only up to half the repetition frequency, and then only by the use of rather sharp filters placed immediately following the *Q* detector. Measurements of -100 dB with respect to the carrier in a 1-Hz band 10 kHz from the carrier require excellent technique.

Similar problems occur in the measurement of the additive noise produced by pulse amplifiers. At Harry Diamond Laboratories some added sensitivity has been obtained by producing another pulse spectrum as similar as possible to that produced by the transmitter and subtracting this.

A suitable device for switching low-level signals is a PIN diode modulator, but with it there is difficulty in obtaining an exact reproduction of the pulse shape produced by a high-power amplifier. On the other hand, the rather large phase perturbation produced by the PIN diode modulator on the leading edge of the pulse is repeated pulse to pulse and produces spectral energy only at multiples of the repetition frequency, where measurements are impossible in any case.

Typical additive-noise measurements made on a variety of FM and CW sources at the Harry Diamond Laboratories[17] and elsewhere are shown in Fig. 14.8. The considerable improvement since 1970 in crystal-oscillator multiplier

*The adjective *residual* sometimes appears in the literature. It is not an apt choice in radar, where the source is usually the element that sets the phase-noise floor.

chains, especially below 5 kHz, can be seen by comparing curve *B* (1970) with curve *C* (1988). Even better performance is to be expected as research in this very important area continues. Although the curves are given to 150 kHz at most, one is often interested in FM noise out to $1/\tau$ (where τ is the pulse length). Solid-state sources, unlike klystrons, have white FM noise at the higher frequencies.[19] This noise folds down in the operation of a pulse doppler radar. The locked source method of measurement, mentioned on page 14.12, can be conveniently altered to measure the total folded noise. The servo is designed to remove noise from the phase detector in the correct proportion to account for the correlation effect. The output of the phase detector is then chopped at the radar's pulse repetition frequency and duty factor. The folding, thus produced, accurately reproduces the radar's demands on its source. The required $1/f^2$ frequency response of the servo is a convenient and stable choice. It is superior to a strictly narrowband loop followed by a shaped amplifier even for CW measurements.

14.6 RECEIVERS

RF Amplification. Although it is apparently attractive, low-noise RF amplification of the received signal has not been extensively employed in CW radar. Transistor amplifiers with excellent noise figures are available to K_u band. Traveling-wave tubes are expensive. In many cases, the determining factor in deciding against a low-noise RF amplifier has been the presence of spillover noise, clutter signals, and signals produced by electronic countermeasures that equal or exceed the noise contributed by conventional front ends.

Few modern receivers have been designed for CW target illuminators since many installations avoid the use of a separate receiver. With space in the nose of an aircraft at a premium, it is usual for airborne missile systems to employ a common antenna for both the tracking radar and the illuminator.[71] In some shipboard weapon systems, the illuminator does not require a receiver since the illuminator antenna is pointed to the direction of the target from information obtained by the tracking radar of the weapon control system rather than have the illuminator track the target itself.

Generation of the Local-Oscillator Signal. To realize adequate signal-to-noise performance, it is customary to perform the first amplification in a CW radar at an intermediate frequency such as 30 MHz. To obtain the necessary coherent local oscillator signal, various types of sidestep techniques are employed. These include modulators, balanced modulators, single-sideband generators (SSG), or phase-locked oscillators.

The SSG is probably the most cumbersome, since the suppression of the carrier and unwanted sidebands is seldom better than 20 dB and filtering must be employed to suppress these signals further. The balanced modulator is much simpler and suppresses the carrier as well as the SSG. The filter needed for further carrier suppression usually suppresses the unwanted sideband to the desired level without added poles. The simple modulator is scarcely less complex than the balanced modulator and requires a sharper filter for the necessary added carrier suppression. Phase locking eliminates the need for high-frequency filtering altogether but requires a skillfully designed servo loop to impress the transmitter's FM noise faithfully on the local oscillator. It may also require a search mechanism to pull in initially. All these methods require the use of an oscillator at the intermediate frequency. The stability required

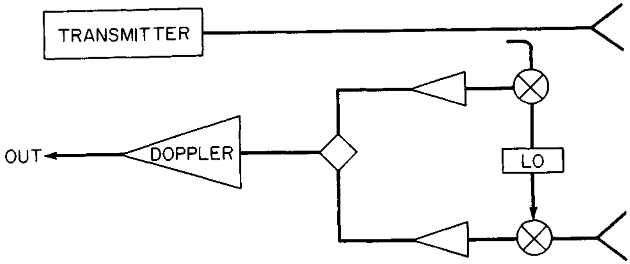


FIG. 14.9 Balanced receiver with a floating LO.

of this oscillator is not excessive since FM is contributed only in the ratio of the IF frequency to microwave frequency.

An alternative approach to the problem of an IF offset is the free-floating local oscillator employed by Harris et al.[13] and by O'Hara and Moore.[8] This has a similarity to the method used to introduce the local oscillator in the Marsh and Wiltshire bridge and requires twin IF amplifiers. The basic diagram is shown in Fig. 14.9. The simple appearance of the figure is deceptive. The local oscillator must normally be positioned by the AFC to hold the signal in the IF bands. As shown, the system folds the doppler frequencies. To avoid this, either quadrature techniques must be employed or a second sidestep be introduced into the reference channel. The latter is unattractive as it destroys the symmetry that may be required to assure uniformity of time delays to cancel the FM noise of the LO. Even in the simplest version the symmetry is far from complete, as the signal channel must handle signals over a wide range of amplitudes while the reference channel carries a signal of uniform amplitude.

IF Amplifier. Traditional low-noise IF amplifiers are usually employed. Because of the levels of clutter signals, ECM, and spillover signals that must be carried by the IF amplifier, it is usual to restrict the gain to no more than 40 dB. This establishes the noise figure and raises the signal to a value where microphonism is less serious without risking levels where saturation and the attendant intermodulation are problems.

Subcarriers. Although doppler filtering may be carried out at slightly higher levels, it is desirable to reject the signal produced by clutter and by spillover at the lowest level possible. Unfortunately, sufficiently high Q's are not available, even in quartz filters, to make it possible to reject clutter at, say, 30 MHz without diminishing the lower doppler frequencies as well.

The simplest method is to mix the signal from the IF amplifier with the signal used in the sidestep. This reduces the spillover signal to dc and the clutter signal to dc and very low frequencies. A multipole filter will suppress those unwanted signals with minor suppression of the very lowest dopplers. Unfortunately, this process folds the spectrum so that incoming targets are indistinguishable from outgoing targets and the random-noise sidebands accompanying each appear in the baseband amplifier. Even if one is prepared to accept the ambiguity, the 3 dB loss in the signal-to-noise ratio (*SNR*) is a matter of concern in a high-power radar.

There are two alternatives, both of which have been extensively employed. The first is a subcarrier band for the doppler intelligence which does not extend to dc but is centered at a frequency where either quartz or electromechanical filters have sufficient Q's to permit sharp filtering. (Values of 0.1 to 0.5 MHz or 1 to 5 MHz are suitable ranges for quartz filters; 0.1 to 0.5 MHz is proper for electromechanical filters.)

The second alternative is quadrature detection.[20] A suitable block diagram for this technique is shown in Fig. 14.10. A single 90° phase shift can be substituted for the +45° and −45° shifts at the constant frequency coming from the oscillator. The plus and minus 45° in the two signal paths are required to maintain a semblance of balance over a wide band of frequencies. A phasor diagram of the system (simplified by omitting the IF sidestep) is shown in Fig. 14.11. If the output from mixer 1 is

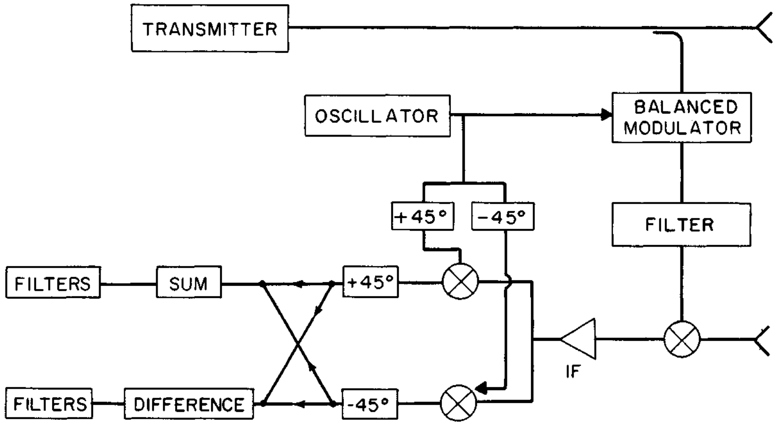


FIG. 14.10 Quadrature receiver.

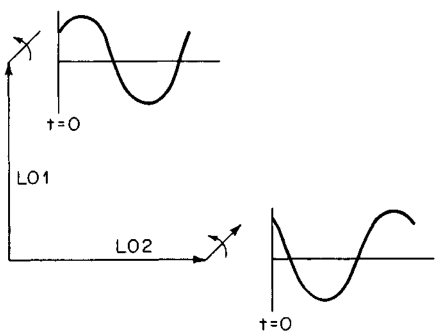


FIG. 14.11 Phasor diagram for a quadrature receiver.

advanced 90° and added to that of mixer 2, the signals will sum in the combiner and disappear in the differencer. A corresponding diagram would show that receding targets reinforce in the differencer and cancel in the combiner.

An advantage of the quadrature system is that the filter bands are completely symmetrical and all filter elements may be identical. Moreover, high-pass filters having steep slopes to reject clutter are somewhat easier to design near dc than they are at low IF. A disadvantage is the requirement to maintain balanced operation over the full range of doppler frequencies in the two second mixers and in the two 45° phase shifters in order to eliminate false targets.

Amplification. After the undesired clutter and spillover signals have been removed, substantial amplification can be achieved either at the second subcarrier frequency or, in the case of folded or quadrature systems, at the doppler frequency itself. It is customary to add additional filtering against the unwanted signals as interstage networks between the stages. The only requirement that must be met is that the combination of the amplification and the total filtering is such that the amplitude of the unwanted signals nowhere approaches the saturation level.

Doppler Filter Banks. Ideally no nonlinear operation occurs in the signal processing and amplification. There is still a coherent signal, and the band narrowing buys decibel for decibel in improved signal-to-noise ratio. Steinberg[21] has shown that, given a fixed doppler band, one pays no penalty in false-alarm rate for subdividing it. In a radar, then, it would be desirable to have the final doppler bandwidth limited only by the time on target. This might be possible in a rapidly scanning radar, but with tracking radars or illuminators the indicated bandwidths would be unrealistically narrow. Moreover, the target itself seldom produces a clearly defined doppler but, rather, a spread of frequencies by the scintillation and glint effects. Bandwidth may also have to be allowed for the coding frequencies which may accompany the doppler, such as those injected by conical scan. A typical circuit for an X-band radar might have a suitable bank of adjacent two-pole filters, each 1000 Hz wide, or an equivalent set of digital filters produced by an FFT.

Following each filter are a detector and a postdetection integrator whose time constant is matched to the time on target or, in the case of a tracker, the demands of the servo data rate. A threshold level is set in the circuitry following each detector; when this is exceeded, a voltage is generated and held until such time as it is read. In acquisition the threshold circuits are normally scanned by some type of readout mechanism. This is fundamentally a computer-type operation.

Doppler Trackers. Doppler filter banks are satisfactory for acquisition and for track-while-scan radars. They are not commonly used in tracking radars or illuminators to improve *SNR* since the use of a doppler tracker (speedgate) is far less complex. The usual speedgate circuit is identical with the AFC circuit in an FM radio. A voltage-controlled oscillator (VCO) is used to beat the signal to be analyzed to a convenient intermediate frequency. A narrowband amplifier at this frequency performs the filtering operation. The VCO is in turn controlled by the output of a discriminator connected to the amplifier. The input to the speedgate can either be the full doppler band, as in the folded or quadrature receiver, or be a subcarrier containing the full doppler intelligence. Although some clutter filtering may take place in the speedgate, earlier removal of unwanted signals is preferable. This is particularly important with the folded receiver in certain airborne situations in which the clutter has a substantial spread because mixer nonlinearities produce harmonics of the unwanted signals that may fall directly on the target signal in the speedgate.

Once in track, the speedgate follows the proper doppler component. The response is limited only by the bandwidth of the servo, which is designed to follow the expected target maneuvers. To acquire track, intelligence may be passed on an open-loop basis from the doppler filter bank, if one is available, or the VCO may have a sawtooth or triangular voltage applied to produce a programmed search. Search is stopped when the output registers the desired target. Coding signals may be employed to aid in the detection and the stopping.

It is usually necessary to restrict the VCO from moving to a frequency that will lock the speedgate on spillover or clutter. With ground-based systems the problem may be simply solved by fixed-limit stops placed on the search voltage. Airborne systems having clutter signals that vary in frequency require more sophisticated solutions.

Constant False-Alarm Rate (CFAR). A constant false-alarm rate in the presence of variable levels of noise is usually a requirement placed on any modern radar. It is very easily achieved in CW radars by the use of filter banks or FFTs. The energy reaching the filter banks is restricted either by automatic gain control (AGC) or, when feasible, by limiting, and the thresholds in the circuitry following the filter banks are properly set with respect to the level in the total band. In a typical setting technique, random noise is injected into the amplifier that drives the filter banks, and each threshold is set to achieve the desired false-alarm rate. The level of noise is then varied and the threshold rechecked. If the limiting is proper, the false-alarm rate should not change. However, target signals in the absence of noise are unaffected, as they do not change the total energy present in the broad doppler spectrum sufficiently to change the AGC level or reach the limiting level. Similar remarks apply to the speedgate as well.

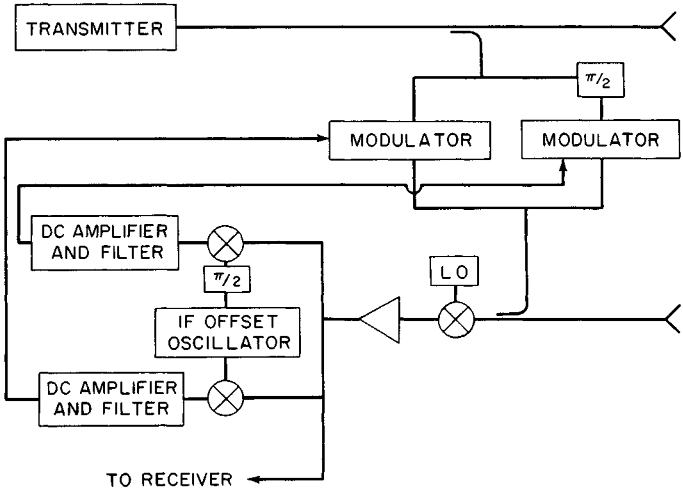
14.7 MINIMIZATION OF FEEDTHROUGH

All major ground-based CW radars have two antennas to minimize spillover. Isolation may be improved further by the use of various absorbers or of an intentional feedthrough path that is adjusted in phase and amplitude to cancel spillover energy. In free space such solutions are all that would be required. When a radar scans across a rough ground plane, however, the energy reflected to the receiver antenna does not remain constant. A dynamic canceler is required. A diagram and description of one such device are given in Ref. 10. Other descriptions are to be found in Refs. 8 and 22.

All dynamic cancelers depend on synthesizing a proper amplitude and phase of a signal taken from the transmitter and using this to buck out the spillover signal. To achieve independence of the servos, the vector is synthesized in orthogonal rectangular coordinates. Figure 14.12 is a typical arrangement for use with a CW radar in which the local oscillator is derived by sidestepping the transmitter. Slight modification[8,13] is needed when the basic radar has the balanced method (Fig. 14.9) of generating the offset required for the first IF amplifier.

The servo amplifiers have response from dc to some frequency well below the doppler band of interest. They respond to the slow variations in the feedthrough signal without damage to the dopplers. For complete details of the mechanical design, see Ref. 8.

Harmer and O'Hara[22] show a variant of the equipment that may be used with a single antenna plus a duplexer. This would be very attractive, especially for an airborne radar that must fit into a small radome. Unfortunately, experience has shown that there is a limit to the transmitter power that may be employed in such



14.12 Feedthrough nulling bridge.

an arrangement. Beyond a modest level of power, the servo is unable to cancel out the −20 dB reflection from the antenna or duplexer sufficiently to prevent receiver degradation.

It should be noted that microwave-feedthrough cancellation is of principal value in preventing saturation and in minimizing the effects of AM noise. Because of the correlation effect, FM noise produced by spillover tends to cancel in the receiver. Near-in AM and FM noise produced by clutter is also beneficially reduced by the spillover servo, since, in nulling out the carrier, it automatically removes both sidebands, whatever their origin, as long as the decorrelation interval is short. Clutter signals from long ranges have both AM and FM noise that is essentially decorrelated, and feedthrough nulling of these signals may increase their deviation by a factor of 2 or their power by a factor of 4. See Eq. (14.3).

14.8 MISCELLANEOUS CW RADARS

There are several small CW radars for applications that require equipment of modest sensitivity. In all these the homodyne technique is employed, the transmitter itself serving as a local oscillator. The transmitter signal reaches the first mixer either by a direct connection or, more frequently, by controlled leakage.

CW Proximity Fuzes. The basic proximity fuze[23,24] is a CW homodyne device whose only range sensitivity is in the rise of the doppler voltage signals as ground is approached or in the behavior of the signal when the antenna pattern intercepts an aircraft. Commonly a single element is used as both oscillator and mixer-detector.

Characteristically, proximity fuzes use a common antenna for transmitting and receiving and hence suffer from a large leakage problem. The situation is tolerable only in the VHF band where the signals returned from the target (terrain or aircraft) are very large. Frequently a projectile body is used as an end-fed antenna although separate transverse dipole or loop antennas have been employed to avoid a null in the forward direction.

The principal problems with the device are those associated with requirements of small size, long shelf life, low cost, and reliability under high acceleration. Because of the very light weight of all solid-state circuitry with integrated components, complex circuits may be built that will allow proximity fuzes to withstand accelerations in excess of 100,000 *g*.

Police Radars. This is a straightforward application of the CW homodyne radar technique. Controlled leakage is used to supply the required LO signal to a single crystal mixer. The amplification takes place at the doppler frequency. At 10,525 MHz, one of the frequencies currently approved by the Federal Communications Commission (FCC), 50 m/h corresponds to 1570 Hz, which is in a convenient range.

A squelch circuit is used to prevent random or noisy signals from reaching the counter. Three amplifier levels relative to the squelch yield suitable gains for the detection of short-, medium-, or maximum-range automobiles. The output signal from the doppler amplifier is clipped, differentiated, and integrated. Each pulse from the differentiator makes a fixed contribution to the integrated signal, and the higher the frequency the greater the output. This dc value actuates a meter or a recording device marked directly in velocity. A tuning fork may be used to calibrate the equipment. Some equipments offer a burst mode which determines the speed of the vehicle before it can be altered.

14.9 FM RADAR

The material to follow is on the homodyne FM radar, i.e., a CW radar in which a microwave oscillator is frequency-modulated and serves as both transmitter and local oscillator. For additional material on FM radars, see Refs. 3 and 6. An excellent introduction to FM in general is contained in Ref. 4, Chap. 12.

There are three approaches to the analysis of this type of radar: the phasor diagram, the time-frequency plot, and Fourier analysis. One should have some facility with each. Perhaps the most useful attack for an FM radar having modest deviation is the phasor diagram. To construct the diagram, a large phasor is drawn to represent the carrier. This is taken as a reference and is considered stationary; higher frequencies are represented by phasors rotating counterclockwise and lower frequencies by phasors rotating clockwise. In applying the phasor method to FM homodyne radar, the instantaneous phase of the local oscillator (i.e., that of the transmitter) is taken as the reference phasor, and the returning signal or signals as the small phasor or phasors. The output from the mixer is proportional to the projection of the small phasor or phasors on the large one.

For example, consider an altimeter with triangular frequency modulation. In its phasor diagram (Fig. 14.13) the small phasor will, except at the turnarounds, swing either clockwise or counterclockwise at a uniform rate. If the swing is short (i.e., the range to the ground is short), then, depending on the phase, either of two situations results: Fig. 14.14*a* or *b*. In Fig. 14.14*a* twice as many cycles of difference frequency will be developed in unit time as in Fig. 14.14*b*. This leads to the so-called critical-distance problem in an FM altimeter. The situation will be covered more fully below; here the interest is in the phasor diagram and what it reveals.

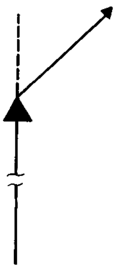


FIG. 14.13 Phasor diagram for an FM-CW radar.

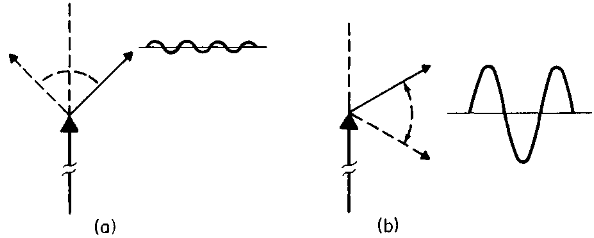


FIG. 14.14 Phasor diagrams showing critical distance

The second method is the drawing of an instantaneous-frequency diagram. In these diagrams a curve is drawn in a time-frequency plane to represent each of the signals of interest. A typical plot, that for a sinusoidally modulated altimeter, is shown in Fig. 14.15. Curve *A* represents the frequency-time history of the transmitter (and local oscillator) and curves *B* and *C* that of returns from two different ranges. Note that the vertical distance between curves (e.g., curves *D* and *E*) yields a heuristic picture of the average frequency behavior of the difference signal from the mixer. This is somewhat naïve. Both the transmitted signal and the returned signal are periodic waves, as is their difference. Hence there cannot be a continuum of difference frequencies; there can be only harmonics of the fundamental modulation frequency. Diagrams such as Fig. 14.15 are most useful when the different frequencies indicated are several multiples of the repetition frequency. In this event, the many harmonic lines act almost like a continuum. Such a diagram would not be useful to discover the step error shown in the phasor diagram above.

Finally, there are mathematical approaches limited originally to those systems

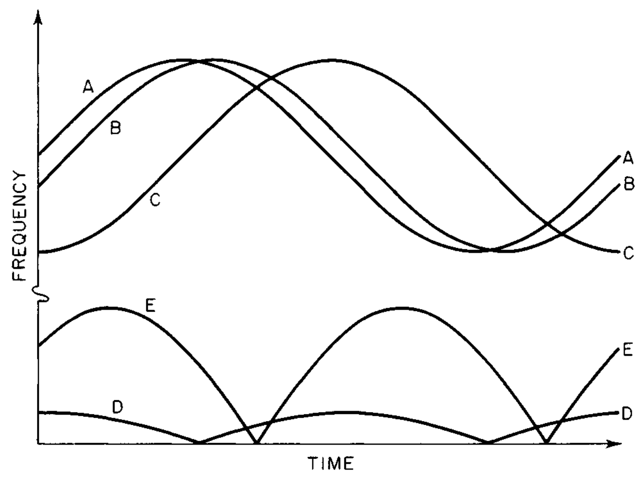


FIG. 14.15 Schematic diagram for a sinusoidally modulated FM.

employing one or more sinusoidal modulations. There exist exact analyses of triangular, sawtooth, dual triangular, dual sawtooth, and combinations of some of these with noise,[25] but heuristic techniques are usually a necessary starting point.

14.10 SINUSOIDAL MODULATION

Suppose one transmits an FM wave of the form

$$\mu_s = U_s \sin\left(\Omega_0 t + \frac{\Delta\Omega}{\omega_m}\sin\omega_m t\right)$$

where ω_m = modulation frequency
Ω_0 = carrier frequency
$\Delta\Omega/\omega_m$ = modulation index

An echo from a point target will have the form

$$\mu_e = U_e\left\{\sin\left[\Omega_0(t-T) + \frac{\Delta\Omega}{\omega_m}\sin\omega_m(t-T)\right] + \phi\right\}$$

where ϕ = arbitrary phase angle produced on reflection
T = time delay of echo

To introduce the effect of doppler we let T be time-dependent: $T = T_0 + 2vt/c$, where v is the velocity of the echoing object and c the velocity of light. After the usual trigonometric manipulation, the difference μ_i takes the form

$$\mu_i = U_i\cos\left[\Omega_0\left(T_0 + \frac{2vt}{c}\right) - \phi + D\cos\omega_m\left(t - \frac{T}{2}\right)\right]$$

$$D = \frac{2\Delta\Omega}{\omega_m}\sin\frac{\omega_m T}{2}$$

The reflection phase ϕ may generally be disregarded and μ_i expanded in a Fourier series.[6]

$$\mu_i = U_i\Bigg(J_0(D)\cos\Omega_0\left(T_0 + \frac{2vt}{c}\right) + \sum_{n\text{ odd}}^{\infty}(-1)^{(n+1)/2}J_n(D)\left\{\sin\left[n\omega_m\left(t - \frac{T}{2}\right)\right.\right.$$
$$\left.\left. + \Omega_0\left(T_0 + \frac{2vt}{c}\right)\right] - \sin\left[n\omega_m\left(t - \frac{T}{2}\right) - \Omega_0\left(T_0 + \frac{2vt}{c}\right)\right]\right\}$$
$$+ \sum_{n\text{ even}}^{\infty}(-1)^{n/2}J_n(D)\left\{\cos\left[n\omega_m\left(t - \frac{T}{2}\right) + \Omega_0\left(T_0 + \frac{2vt}{c}\right)\right]\right.$$
$$\left.)\cos\left[n\omega_m\left(t - \frac{T}{2}\right) - \Omega_0\left(T_0 + \frac{2vt}{c}\right)\right]\right\}\Bigg)$$